MATHEMATICS: PAPER I

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 8 pages and an Information Sheet. Please check that your paper is complete.

2.   Read the questions carefully.

3.   Answer all the questions.

4.   Number your answers exactly as the questions are numbered.

5.   You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6.   Round off your answers to one decimal digit where necessary.

7.   All the necessary working details must be clearly shown.

8.   It is in your own interest to write legibly and to present your work neatly.

9.   Please hand in this question paper.

SECTION A

QUESTION 1

Solve for x:

(a) $\dfrac{x}{3} = \dfrac{3}{x}$ $x \neq 0$ (2)

(b) $(2x-1)(5x+4) = (x+1)(x+5) - 3$ (4)

(c) $x(x+2) > 3$ (4)

(d) $\log_x 9 = -2$ (3)

[13]

QUESTION 2

(a)   Write down the nth term of the sequence: $\dfrac{1}{4}$; $\dfrac{4}{9}$; $\dfrac{9}{16}$; $\dfrac{16}{25}$; ...   (1)

(b)   Given the geometric sequence: $\dfrac{3}{8}$; $\dfrac{3}{4}$; $\dfrac{3}{2}$ ...

     (1)   Determine the value of the 10th term.   (3)

     (2)   Determine which term has a value of 12 288.   (4)

(c)   An arithmetic series has 21 terms. The first term is 3 and the last term is 53. Find
the sum of the series.   (3)

[11]

QUESTION 3

(a)   Given: $f(x) = -2(x+4)^2 + 3$

Determine the turning points of:

(1) $y = f(x) - 1$ (3)

(2) $y = f(x+1)$ (2)

(3) $y = 4f(x)$ (2)

(b)   Refer to the figure showing a sketch of $y = g(x)$ and $y = g^{-1}(x)$ where $g(x) = b^x$.

T is the point of intersection of the two graphs. The graph of g contains the point K$(-1; 2)$.
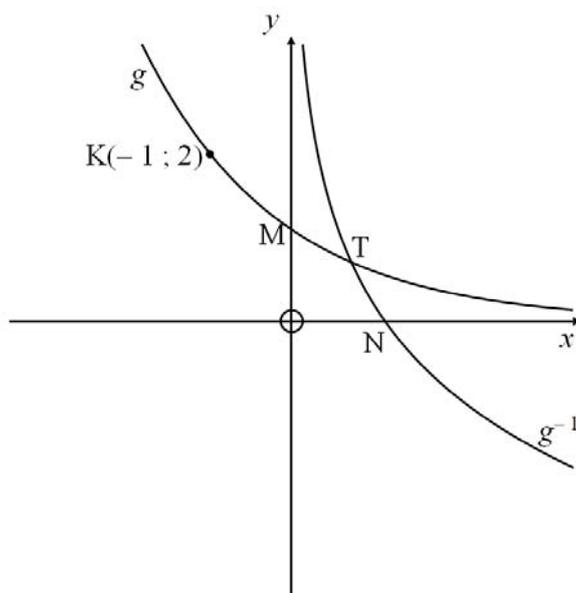
(1)   Show that $b = \dfrac{1}{2}$.   (1)

(2)   Determine the equation of $g^{-1}(x)$ in the form $g^{-1}(x) = \ldots$ (2)

(3)   The x-coordinate of T is 0,64 (correct to 2 decimal digits). Give the y-coordinate of T (correct to 2 decimal digits).   (2)

(4)   Write down the value(s) of x for which $g^{-1}(x) > 0$.   (2)

(5) $h(x)$ is the reflection of $g^{-1}(x)$ about the y-axis. Give the equation of $h(x)$, stating the domain.   (2)

[16]

QUESTION 4

(a)   Shazia invested a sum of money that doubled in value after 5 years. Determine the annual compound interest rate that was used. Give your answer as a percentage correct to one decimal digit. (4)

(b)   Jo bought a plasma TV which was priced at R13 499. He paid R1 000 deposit and signed a contract for the balance to be paid in 24 monthly instalments of R635,37 where interest was charged on a simple interest basis. Calculate the interest rate per annum, to the nearest percent. (5)

(c)   (1)   Sam can afford to pay R6 500 per month as a repayment of a loan. The bank charges 9,5% p.a. compounded monthly on a loan to be repaid in equal monthly instalments over 15 years. Calculate the loan amount that Sam can expect to receive. Instalments start one month after the granting of the loan. (3)

      (2)   Suppose that Sam decided to take a loan of R550 000 and pay R6 500 per month. The interest remains at 9,5% p.a. compounded monthly. Calculate how many years and months it will take for Sam to repay the loan. Give your answer correct to the nearest month. (6)

[18]

QUESTION 5

(a)   Given: $f(x) = \dfrac{x^2}{4}$, determine $f'(x)$ from first principles. (5)

(b)   Find $\dfrac{dy}{dx}$ for $y = \dfrac{15x^2 + x - 2}{3x - 1}$ $x \neq \dfrac{1}{3}$ (3)

(c)   Given: $f(x) = \sqrt{x} + \dfrac{1}{x^2} - 3$ $x > 0$

Evaluate $f'(4)$. (5)

(d)   Given: $f(x) = x^3 - 3x^2 + kx + 8$ where k is a constant. The graph has a turning point at $x = 1$. Find the value of k. (4)

[17]

75 marks

SECTION B

QUESTION 6

Refer to the figure showing the graphs of $f(x) = \dfrac{x^2}{2} - \dfrac{7x}{2} + 3$ and $g(x) = -x + 6$.

D is the x-intercept of both f and g.
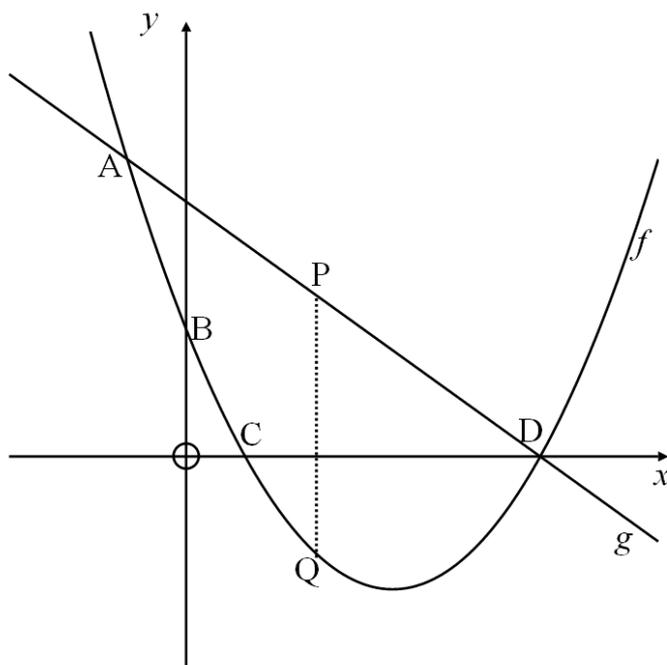
(a)   Determine the coordinates of A.   (5)

(b)   Write down the values of x for which $f(x) \geq g(x)$.   (2)

(c)   Determine the equation of the tangent to f at C, an x-intercept of f.   (8)

(d)   PQ⊥CD, P on g and Q on f such that P lies between A and D. Determine the maximum length of PQ.   (6)

[21]

QUESTION 7

Zanele plays netball and chess every Saturday.

Let x be the number of hours that she spends playing netball and y be the number of hours she spends playing chess.

She has some restrictions on the amount of time that she does activities.

The diagram below shows all the constraints defining the feasible region.
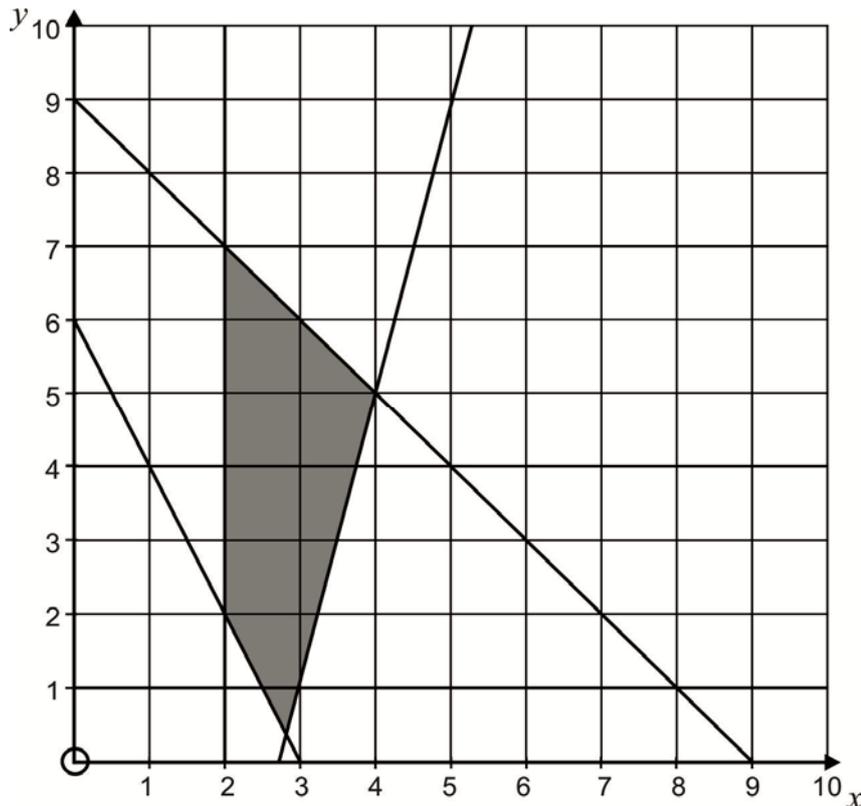
One of the constraints is $y \geq 4x - 11$.

(a) Determine the other inequalities involved in defining this feasible region.   (6)

(b) Suppose Zanele plays netball for $2\frac{1}{2}$ hours. Determine the interval of hours available for chess.   (3)

(c) Zanele's medical aid awards points for various activities. For netball and chess, it awards $W = 5x + 2y$ points. Determine the number of hours that Zanele should spend on netball and chess in order to achieve the highest number of points.   (3)

(d) Hence determine the highest number of points that she can score.   (2)

[14]

QUESTION 8

DataSuppliers has monthly charges for data transfer as follows:

* The first gigabyte (GB) of data transfer costs R30.
* Each GB of data transfer after this costs R1,50 less than the previous one.
* When the cost of a GB of data transfer reaches zero, any additional data transfer during that month is free.

(a)   Calculate the cost of the fifth GB of data transfer.   (3)

(b)   Determine how many GB are transferred within the month before downloads become free.   (3)

(c)   Calculate the cost of transferring **a total** of 17 GB of data.   (3)

(d)   A customer paid the maximum charge for data transfer. Determine this amount.   (4)

[13]

QUESTION 9

(a)   The radioactive decay of a substance is given by the formula:

$m(t) = 500(0,92)^t$ where $m(t)$ is the mass(in grams) of the radioactive substance and t is its age in years.

(1)   Write down the initial mass of the radioactive material.   (1)

(2)   Write down the percentage by which the radioactivity decreases each year.   (1)

(3)   Determine the mass of the substance after 50 years.   (2)

(4)   Determine the least number of years it takes for the substance's mass to be less than one gram. Give your answer to the nearest year.   (3)

(b)   Given: $f(x) = \dfrac{x}{2}$ when x is rational

but $f(x) = x^2$ when x is irrational

Evaluate $f\left(\sqrt{4}\right) + f\left(\sqrt{8}\right)$.   (3)

(c)   Given: $f(x) = \dfrac{2}{x-3} + 1$

(1)   Write down the equation of both asymptotes of the graph $y = f(x)$.   (2)

(2)   Determine the lines of symmetry of $y = f(x+5)$.   (3)

[15]

QUESTION 10

Refer to the figure, showing a sketch of $y = f(x)$.
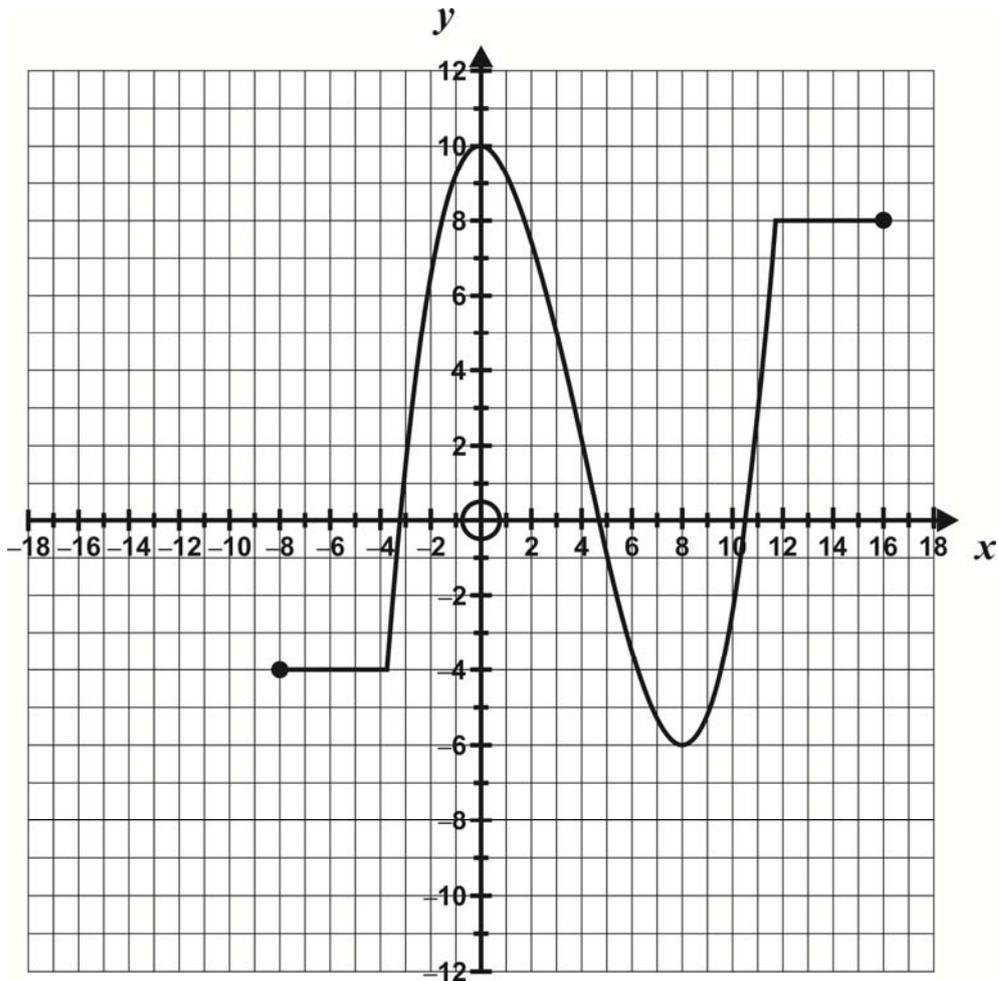
(a)   State the domain and range of f.   (4)

(b)   A function g is said to be an odd function if $g(-x) = -g(x)$ for all values of x in its domain. Explain why an odd function is always symmetrical about the origin. That is, if $(a;b)$ is on the graph of g, then $(-a;-b)$ is also on the graph of g.   (4)

(c)   It is possible to translate the graph of f above so that the new graph is the graph of an odd function. Determine p and q so that $g(x) = f(x-p)+q$ is an odd function.   (4)

[12]

75 marks

Total: 150 marks